3.6 Measuring Resistance: The Wheatstone Bridge

. The Wheatstone bridge is used to measure resistances in the range 1Ω to $1 M\Omega$

. Accuracies of the order of $\pm 0.1\%$ are possible commercially.

. The Wheatstone Bridge Circuit:

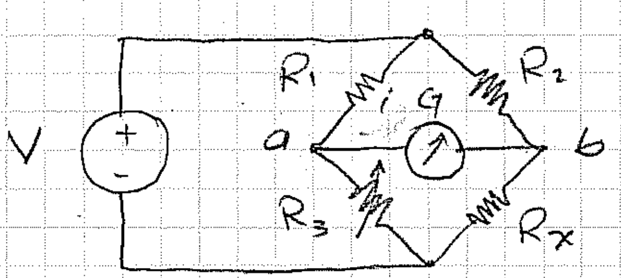


Fig. 3.26.

Adjust R_3 until there is no current in the Galvanometer. R_x is given by:

$$R_x = \frac{R_2}{R_1} R_3 \qquad (3.33)$$

When the bridge is balanced, the current in the galvanometer (G) is zero. So points a and b are at the same potential. So:

$$i_3 R_3 = i_x R_x \qquad (3.36)$$

and $$i_1 R_1 = i_2 R_2 \qquad (3.37)$$

Since current in G is zero, by KCL at nodes a and b we have:

$i_1 = i_3$ and $i_2 = i_x$

if we replace in 3.36 and 3.37 and divide the two equations we obtain:

$$\frac{R_3}{R_1} = \frac{R_x}{R_2} \Rightarrow R_x = \frac{R_2}{R_1} R_3$$

If the values of R_1, R_2 and R_3 are known, we can determine R_x.

Assessment Problem 3.7:

For the bridge circuit shown in Fig. 3.26 $R_1 = 100\Omega$, $R_2 = 1000\Omega$, $R_3 = 150\Omega$. $v = 5V$

a) What is R_x?

$$\frac{R_x}{R_2} = \frac{R_3}{R_1} \Rightarrow R_x = \frac{150}{100} \times 1000 = 1500\Omega$$

b) Can the bridge be left in the balanced state without exceeding the power dissipation of the resistors given at 250mW?

We need to calculate the currents i_1 and i_2 $(= i_x)$.

$$i_2 = i \frac{R_{eq}}{R_2 + R_x} \qquad R_{eq}\, i = v = 5V \Rightarrow$$

$$i_2 = \frac{5}{2500} = 2 \times 10^{-3} A = 2mA$$

$$P_2 = R_2 i_2^2 = 1000 \times (2\times 10^{-3})^2 = 4\times 10^{-3} = 4\ mW$$

which is way below the 250 mW limit.
Similarly for other resistors:

$$P_x = R_x i_x^2 = 1500 \times (2\times 10^{-3})^2 = 6\ mW$$

The power dissipation of R_1 and R_3 will be calculated using the voltage division rule.

$$v_1 = v \frac{R_1}{R_1+R_3} = 5 \times \frac{100}{100+150} = 2\ V$$

$$v_2 = v \frac{R_2}{R_1+R_3} = 5 \ \frac{150}{100+150} = 3\ V$$

(or $v_2 = v - v_1$ (KVL)!).

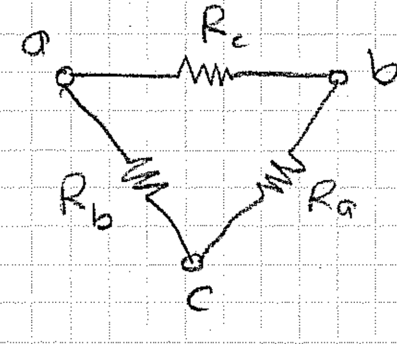
$$P_1 = \frac{v_1^2}{R_1} = \frac{4}{100} = 0.04\ W = 40\ mW$$

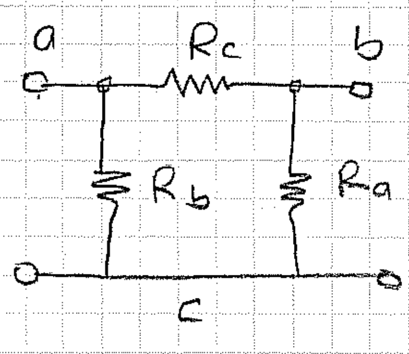
$$P_2 = \frac{v_2^2}{R_2} = \frac{9}{150} = 0.06\ W = 60\ mW$$

So the power dissipation in all resistors is $\ll 250\ mW$, and thus we can leave the bridge in the balanced state for an extended period of time.

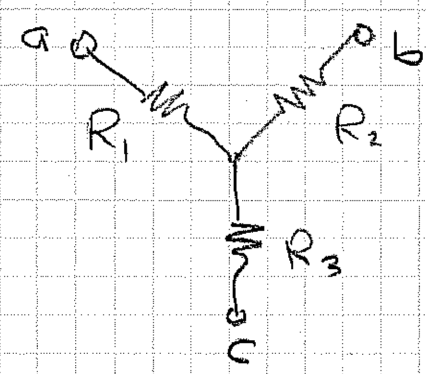
3.7 Delta-to Wye (Δ-Y) or Pi-to-Tee (Π-T) Equivalent Circuits.

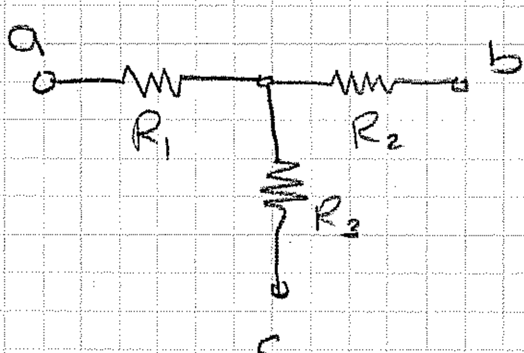
A Δ configuration can be viewed as a Π:





A Y configuration can be viewed as a T:





For a Δ (or Π) configuration to be equivalent to a Y (or T). The resistances seen (measured) between any two terminals on the Δ must be equivalent to the corresponding in the Y:

$$R_{ab(\Delta)} = R_{ab(Y)}$$

$$R_{ab(\Delta)} = R_c // (R_a + R_b)$$

$$R_{ab}(Y) = R_1 + R_2$$

So: $$\frac{R_c(R_a+R_b)}{R_a+R_b+R_c} = R_1 + R_2 \quad (3.41)$$

Similarly:

$$R_{bc} = \frac{R_a(R_b+R_c)}{R_a+R_b+R_c} = R_2 + R_3 \quad (3.42)$$

and $$R_{ca} = \frac{R_b(R_a+R_c)}{R_a+R_b+R_c} = R_1 + R_3 \quad (3.43)$$

Solve for R_1, R_2 and R_3 in terms of R_a, R_b and R_c to obtain the equations of a $Y-\Delta$ transformation $(T-\pi)$:

$$R_1 = \frac{R_b R_c}{R_a+R_b+R_c} \quad (3.44)$$

$$R_2 = \frac{R_a R_c}{R_a+R_b+R_c} \quad (3.45)$$

$$R_3 = \frac{R_a R_b}{R_a+R_b+R_c} \quad (3.46)$$

For the $\Delta-Y$ transformation we solve 3.41 → 3.43 for R_a, R_b, and R_c in terms of R_1, R_2 and R_3:

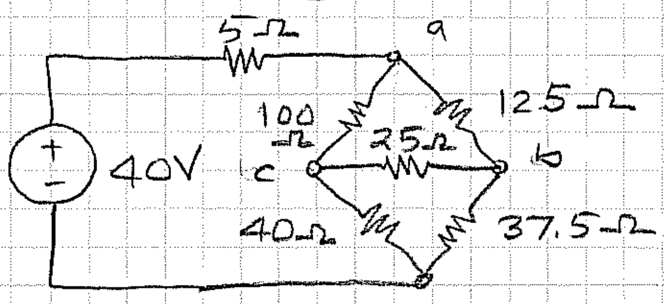
$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \qquad (3.47)$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \qquad (3.48)$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \qquad (3.49)$$

Example 3.7:

Use a Δ-Y transformation to find the current and power supplied by the source in the following circuit:



Solution:

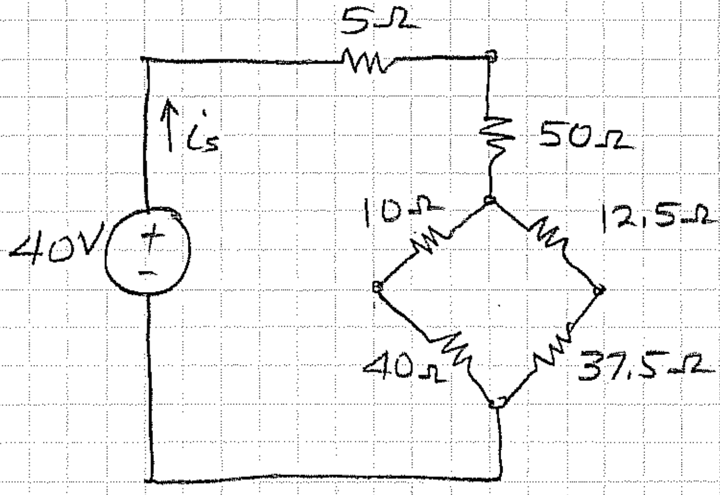
Use equations 3.44 to 3.46 to transform the top Δ to a Y:

$$R_1 = \frac{100 \times 125}{100+125+25} = 50\Omega$$

$$R_2 = \frac{125 \times 25}{250} = 12.5$$

$$R_3 = \frac{100 \times 25}{250} = 10\Omega$$

The circuit following the transformation becomes:



R_{eq} "seen" by the source is:

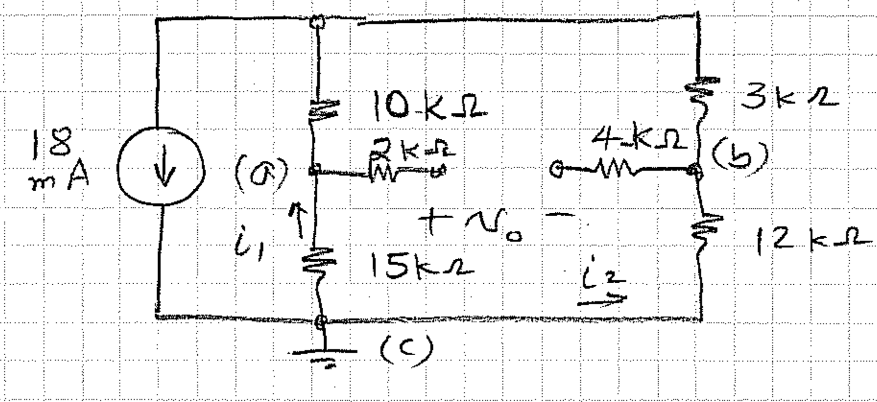
$$R_{eq} = 5 + 50 + \frac{50 \times 50}{100} = 80\Omega$$

$$i_s = \frac{40}{80} = 0.5A$$

$P_s = 40 \times 0.5 = 20$ W. is delivered to the circuit.

Problem 3.26

Find voltage v_0 in the following circuit:



Check whether bridge is balanced.

$$\frac{10}{15} = 0.667 \neq \frac{3}{12} = 0.25$$

The bridge is not balanced.

By KVL $v_o = v_{ac} - v_{bc}$, so let us calculate v_{ac} and v_{bc}. To do that we need to determine the current in each branch. We will do that using the current divider rule:

$$i_1 = 18 \times \frac{15}{15+25} = 6.75 \text{ mA}$$

$$i_2 = 18 \times \frac{25}{15+25} = 11.25 \text{ mA}$$

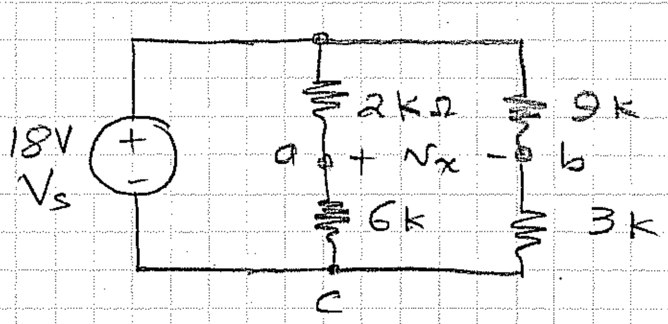
$$v_{ac} = -i_1 \times 15 = -6.75 \times 15 = -101.25 \text{ V}$$

$$v_{bc} = -i_2 \times 12 = -11.25 \times 12 = -135 \text{ V}$$

$$v_o = v_{ac} - v_{bc} = -101.25 + 135 = 33.75\text{ V.}$$

Problem 3.27

a) Find the voltage v_x in the circuit shown below:



$$v_x = v_{ac} - v_{bc}$$

$$v_{ac} = 18 \times \frac{6}{18} = 6\text{ V}$$

$$v_{bc} = 18 \times \frac{3}{12} = 4.5\text{V}$$

$$v_x = 6 - 4.5 = 1.5\text{V.}$$

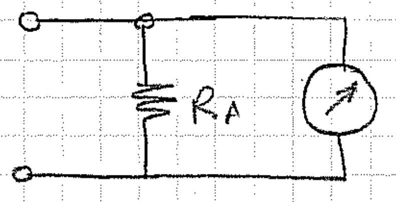
Note: Bridge is not balanced!

b) Find v_x as a function of V_s:

$$v_{ac} = V_s \times \frac{1}{3} \qquad v_{bc} = V_s \times \frac{1}{4}$$

$$v_x = V_s\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{V_s}{12}$$

3.39 50 mV, 1 mA d'Arsonval movement.
What is the largest full-scale reading if the shunt resistors available have a 0.5 W rating?



At full scale the voltage drop across the ammeter is 50 mV,

The power dissipated in R_A is:

$$P = \frac{(50\times10^{-3})^2}{R_A} = 0.5 \Rightarrow$$

$$R_A = \frac{(50\times10^{-3})^2}{0.5} = 5\times10^{-3} = 5\,m\Omega$$

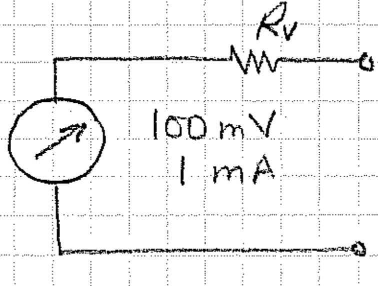
Note that largest full-scale corresponds to the smallest R_A!

The current in the ammeter is the current in R_A (very nearly) because $R_A\ (= 5\,m\Omega) \ll R_{mov} = 50\,\Omega$

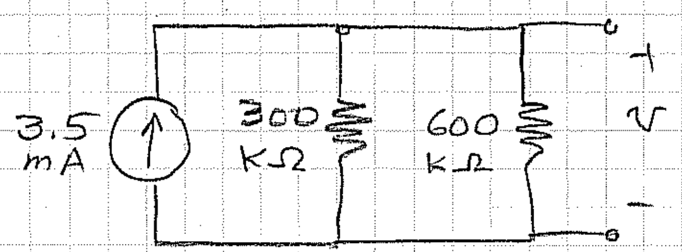
So the full scale reading is

$$I_{FS} = 50\times10^{-3} / (5\times10^{-3})$$
$$= 10\,A.$$

Problem 3.40: The voltmeter shown has a full-scale reading of 800 V, the meter movement is rated at 100 mV and 1 mA.



What is the error if it is used to measure the voltage v in the following circuit?



The voltage is calculated as:

$$v = 3.5 \times \frac{300 \times 600}{300 + 600} = 3.5 \times 200$$
$$= 700\,V$$

The resistance of the meter is obtained from KVL applied on the meter at full scale:

$$800 = R_v \times 1 \times 1 + 100 \times 10^{-3} \Rightarrow$$
$$R_v \simeq \frac{800}{1} = 800\,k\Omega$$

when inserted in parallel with the given circuit the measured voltage v_m would be read as:

$$v_m = 3.5 \times \frac{200 \times 800}{200 + 800} = 3.5 \times 160$$
$$= 560 \text{ V}$$

The error is defined by:

$$\text{error} = \left(\frac{\text{measured}}{\text{true}} - 1\right) \times 100\%$$
$$= \left(\frac{560}{700} - 1\right) \times 100 = -20\%$$

Note:

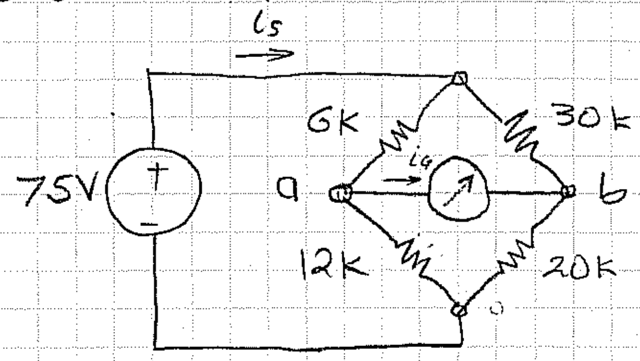
Obviously, the meter is not appropriate to measure the voltage is the circuit. It will be able to measure voltages across resistances that should be much smaller than 800 kΩ. For example if the resistances were 30 kΩ and 60 kΩ and the current source 35 mA. The voltage v would still be 700 V.

But the measured voltage would be:

$$v_m = 35 \times \frac{20 \times 800}{820} = 682.9 \text{ V}$$

The error would then be −2.44% !

3.51



Find the detector current in the bridge (i_G) if the voltage drop across the detector (G) is negligible.

Solution:

Since $\frac{6}{12} = \frac{1}{2} \neq \frac{30}{20} = 1.5$, the bridge is not balanced. Nevertheless points a, and b are at the same potential because the voltage drop across the galvanometer is zero!

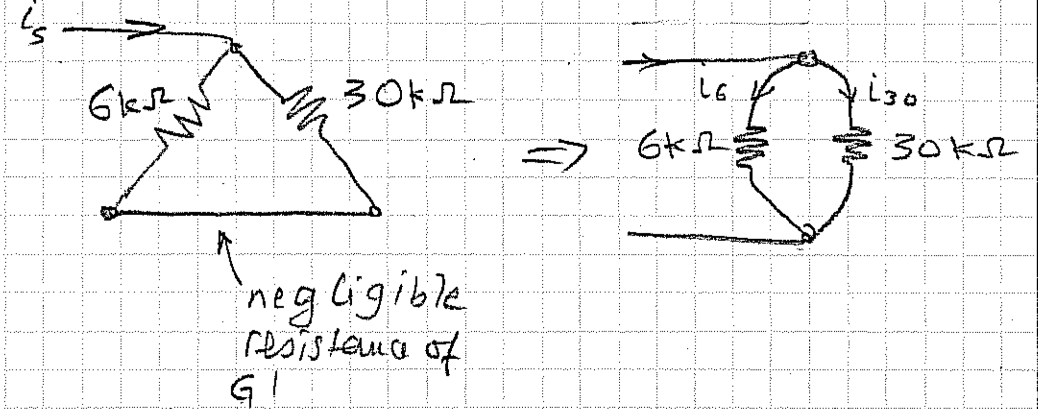
Since points a and b are at the same potential, I can short them! Then the equivalent resistance is obtained as follows:

$$R_{eq} = (6 // 30) + (12 // 20)$$
$$= 5 + 7.5 = 12.5\ k\Omega$$

The current from the source is:

$$i_s = \frac{75}{12.5} = 3\ mA$$

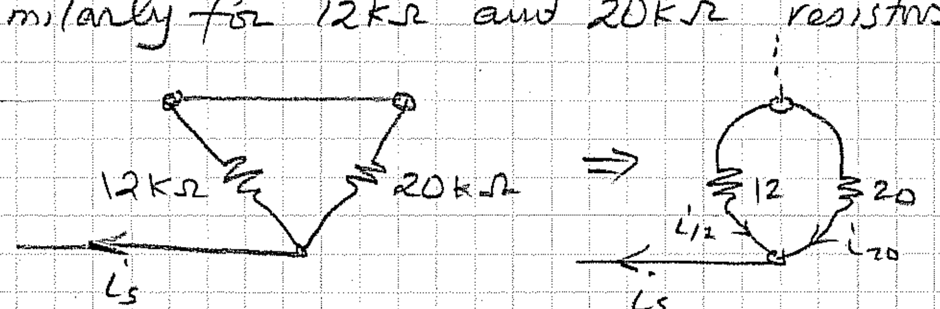
Since points a and b are at the same potential, then the currents in the 6 and 30kΩ resistors can be found using the current divider rule



So $\quad i_6' = i_s' \dfrac{30}{36} = 3 \times \dfrac{30}{36} = 2.5\,mA$

$i_{30}' = 3 - 2.5 = 0.5\,mA$

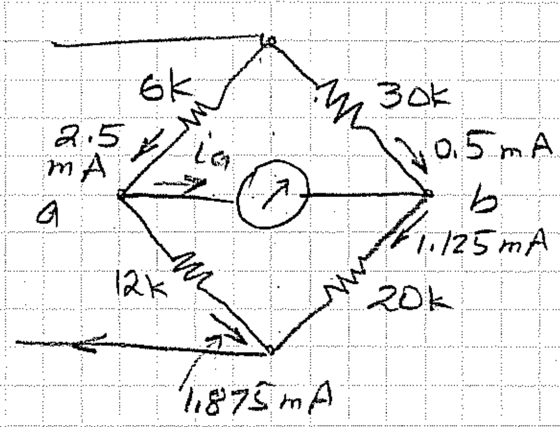
Similarly for 12kΩ and 20kΩ resistors:



$i_{12}' = i_s' \dfrac{20}{32} = 3 \times \dfrac{20}{32} = 1.875\ mA$

$i_{20}' = i_s' \dfrac{12}{32} = 3 \times \dfrac{12}{32} = 1.125\,mA$

Now if we go back to the original circuit:



i_G can be found from KCL at node a:

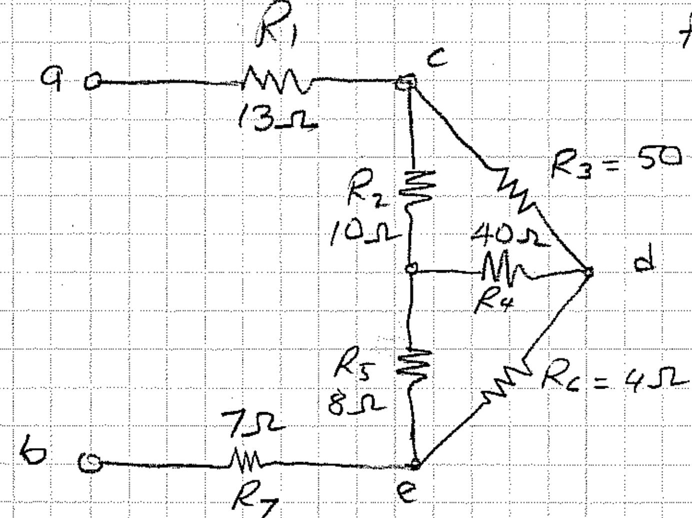
$i_G + i_{12} = i_6 \Rightarrow$

$i_G = i_6 - i_{12} = 0.625$ mA

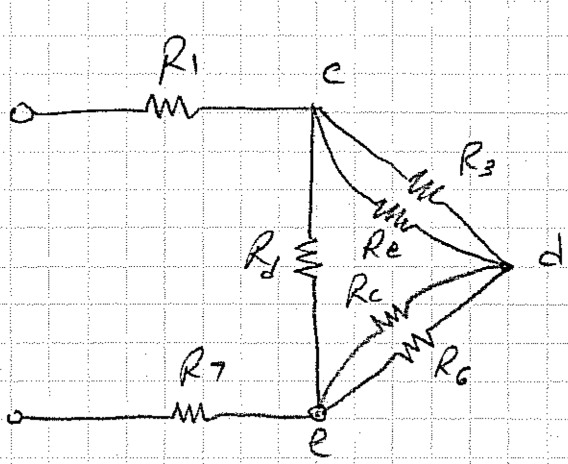
The same answer is obtained with KCL at node b.

Problem 3.56

Find R_{ab}.



Hint: Do a Y→Δ transformation involving R_2, R_4 and R_5, the following equivalent circuit will emerge:



Using equations 3.47 to 3.49:

$$R_c = \frac{R_2R_4 + R_4R_5 + R_5R_2}{R_2} = \frac{800}{10} = 80\,\Omega$$

$$R_d = \frac{R_2R_4 + R_4R_5 + R_5R_2}{R_4} = \frac{800}{40} = 20\,\Omega$$

$$R_e = \frac{R_2R_4 + R_4R_5 + R_5R_2}{R_5} = \frac{800}{8} = 100\,\Omega$$

$$R_{ab} = R_1 + R_{ce} + R_7$$

with $R_{ce} = R_d \,//\, \left((R_3 \,//\, R_e) + (R_6 \,//\, R_c) \right)$

$$= 20 \,//\, \left(\frac{50 \times 100}{150} + \frac{4 \times 80}{84} \right)$$

$$= 20 \,//\, 3.333 + 3.8095$$

$$= \frac{20 \times 7.142857}{27.142857} = 5.2632\,\Omega$$

$$R_{ab} = 1 + 5.26 + 7 = 13.26\,\Omega$$